# Convexity

What makes a problem easy to solve

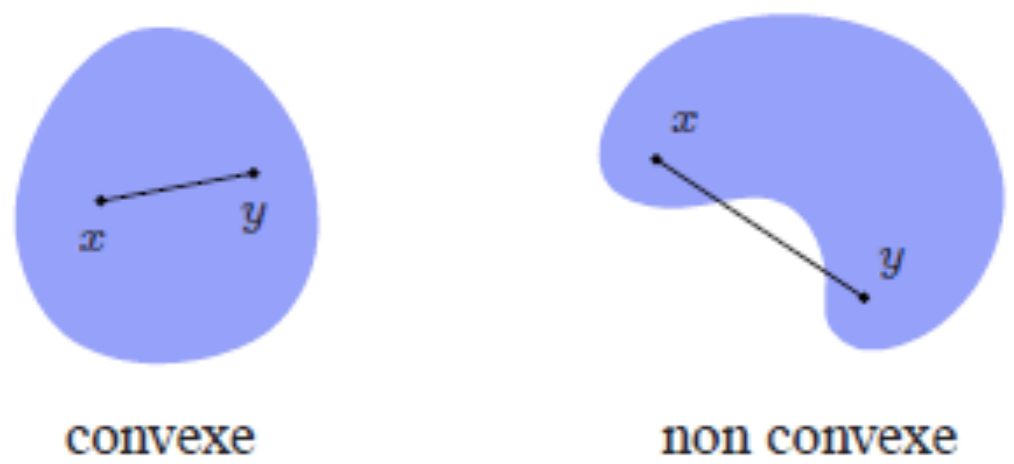
**Short talk : 07/02/2022**

# What is a easy problem?

- *« …in fact, the great watershed in optimization isn't between linearity and nonlinearity, but convexity and nonconvexity. »* - R. Tyrrell Rockafellar, in SIAM Review, 1993

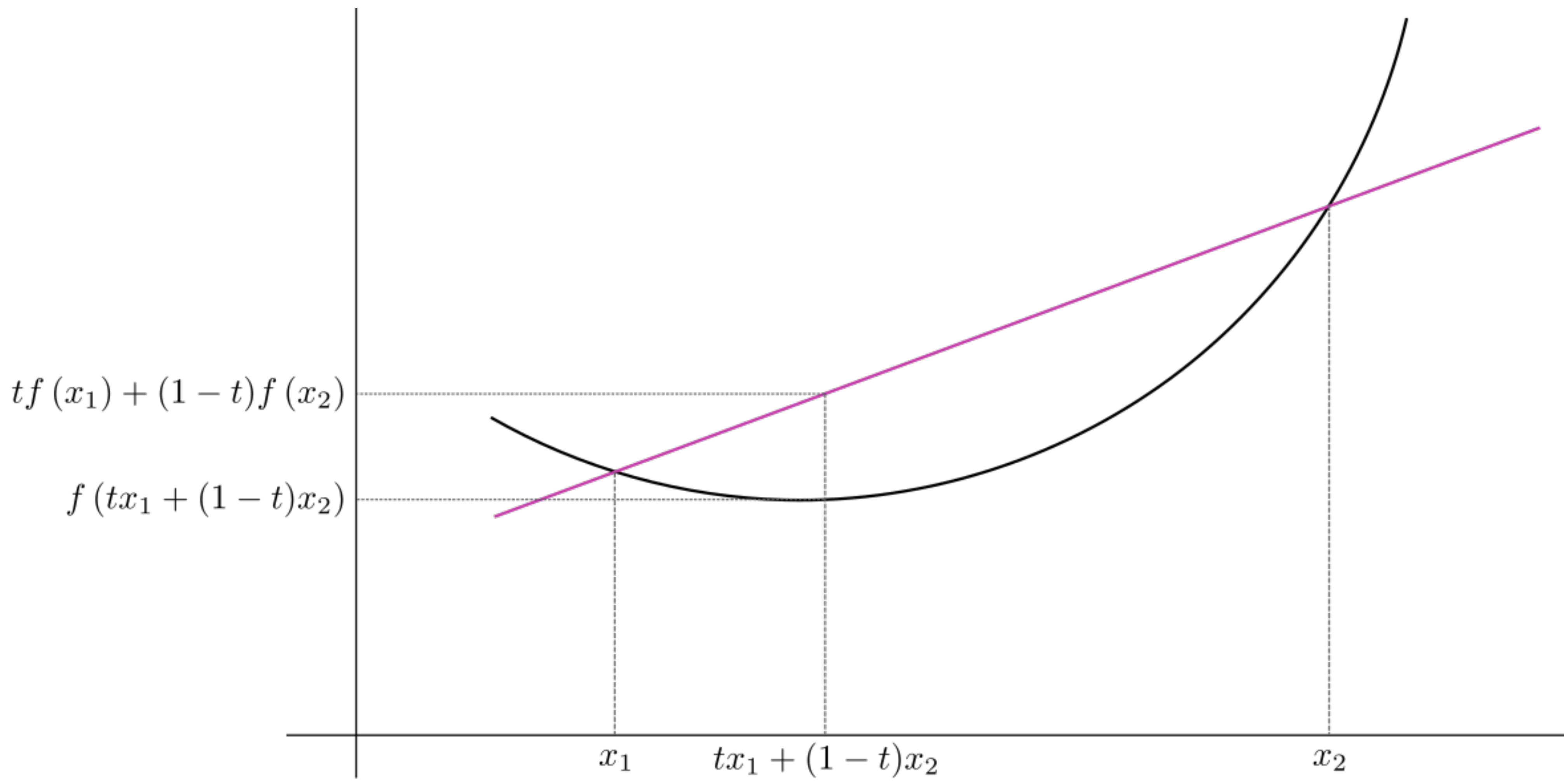
- Two conditions for an optimization problem to be convex: convex objective function & convex feasible set.

# Convex set




Given any two points in the subset, the subset contains the whole line segment that joins them.

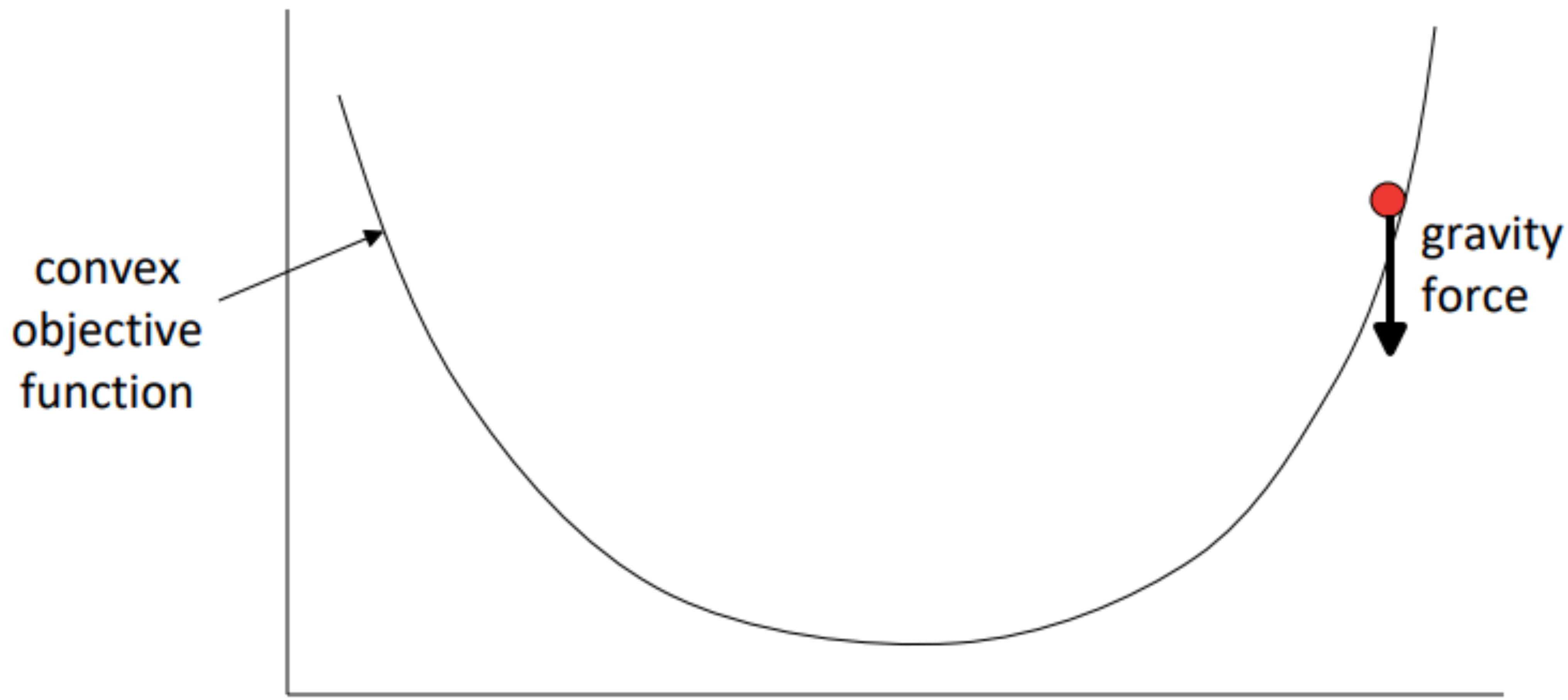
# Convex objective function




**Examples:** $e^x, \quad x^2$

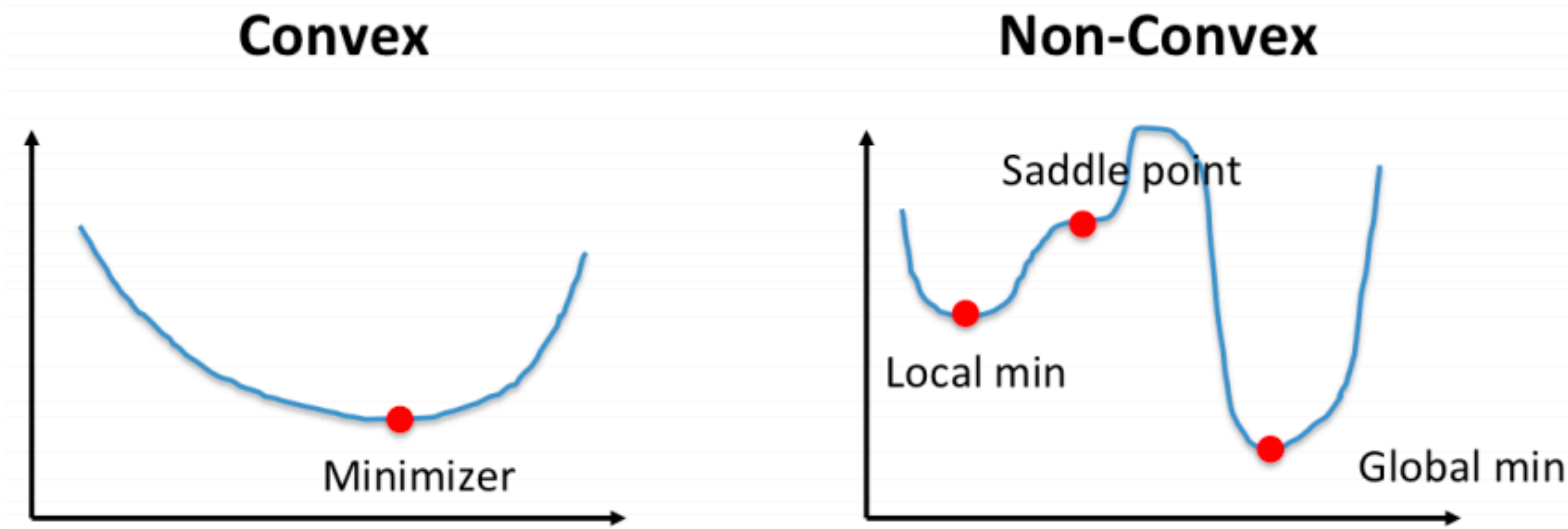
**For one variable function : f (strictly) convex iif f"(x) (strictly) positive.**

Let gravity do the hard work for you:



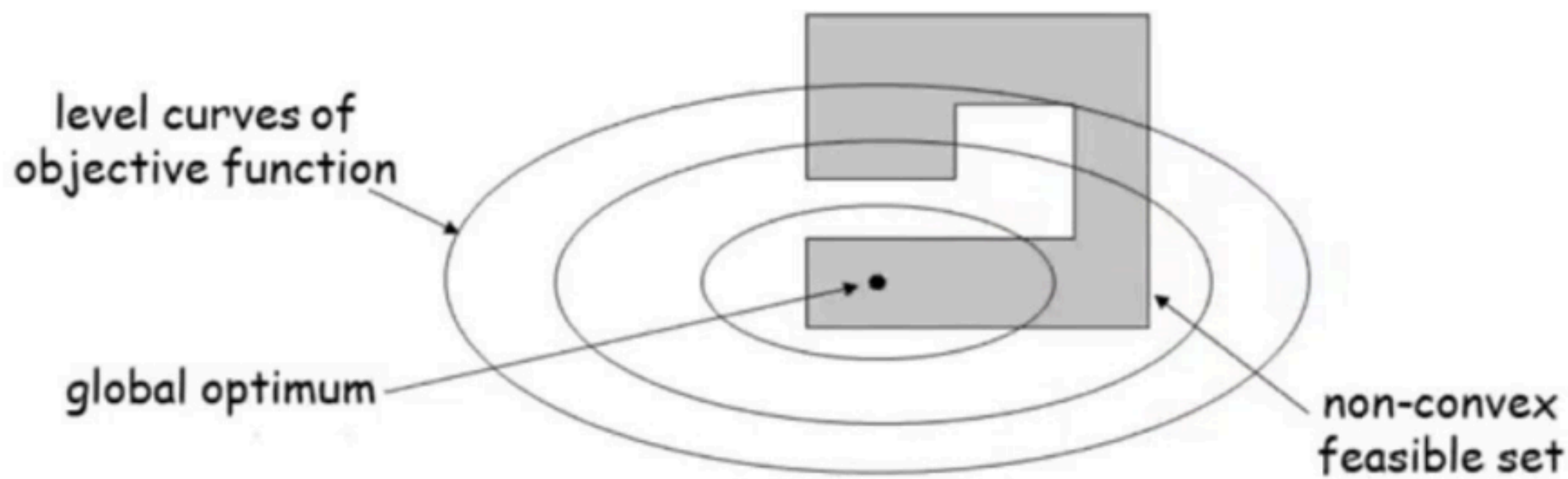

More formally, let gradient descent do the hard work for you.

In convex problems, gravity takes you to global minimum.




In non-convex problems, gravity may take you to sub-optimal locations.

# Why do we need the feasible set to be convex as well?

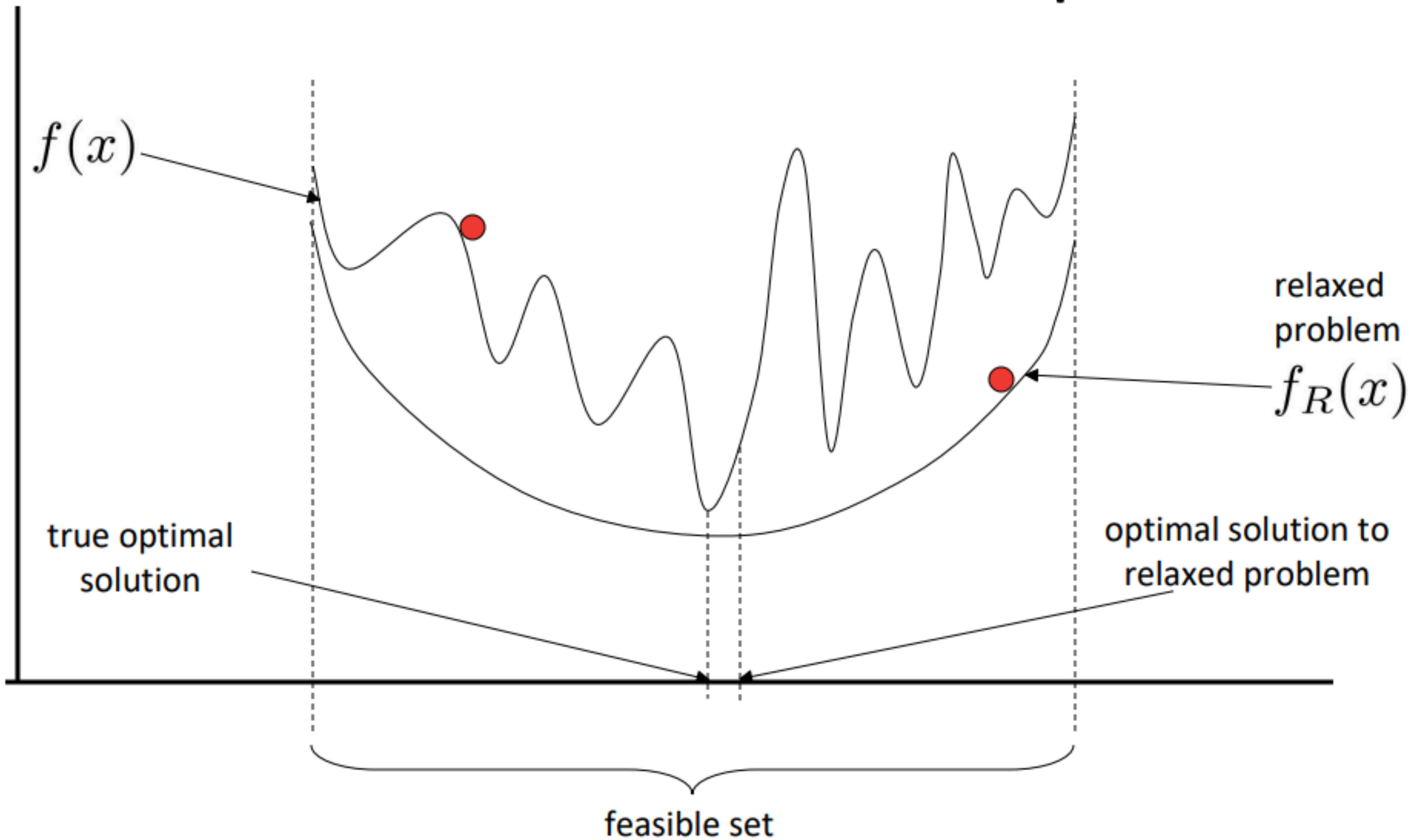



We may get stuck in a local optimum if we simply "follow gravity".

# Key properties of Convex problems

- Every local minimum is a global minimum.
- The optimal set is convex.
- If the objective function is *strictly* convex, then the problem has at most one optimal point.

# The relaxation technique




Try to approximate your original difficult problem with another one (the so called relaxed problem) which is easier to solve:

- By dropping some constraints (so that the enlarged set is convex).

- By modifying the objective function (so that the new function is convex).