Optimal Transport with Adaptive Regularisation

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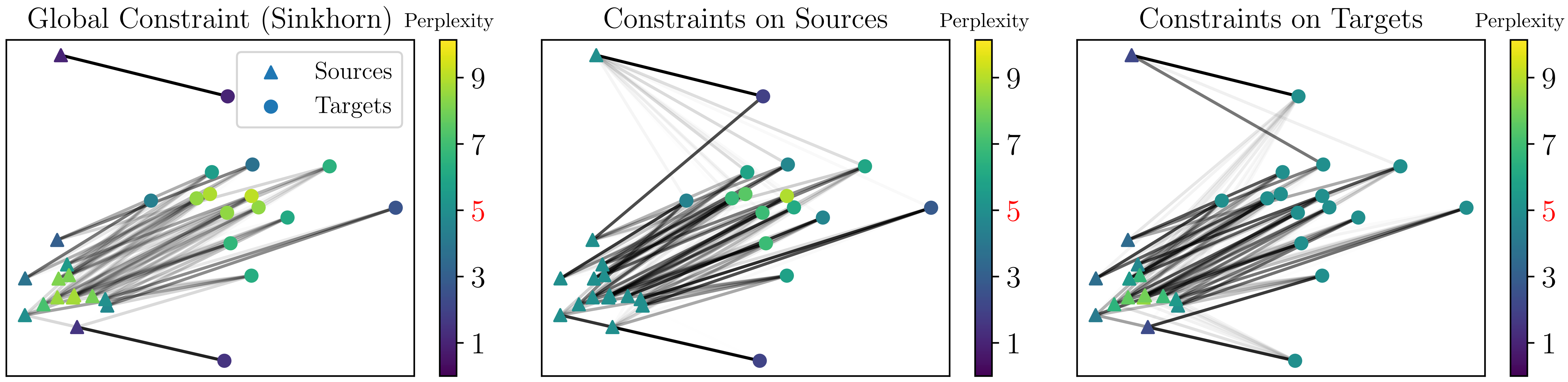


Regularising OT with a strictly convex term leads to **reduced numerical complexity** and **more diffuse OT plans**. Current formulations of regularised OT are equivalent to imposing a **global constraint on the OT plan**.

Regularised OT

$$\min_{\mathbf{P} \in \Pi(\boldsymbol{a},\boldsymbol{b})} \langle \mathbf{P}, \mathbf{C} \rangle \quad \text{s.t.} \quad \sum_i \psi(\mathbf{P}_{i:}) \leq \eta \,.$$

$\psi : \mathbb{R}^{N_s} \to \mathbb{R}$ is any strictly convex regulariser.

$$\begin{cases} \psi_{\mathrm{KL}}(\mathbf{p}) = \langle \mathbf{p}, \log \mathbf{p} - 1 \rangle & \Rightarrow \textbf{Entropic OT} \\ \psi_2(\mathbf{p}) = \frac{1}{2}\|\mathbf{p}\|_2^2 & \Rightarrow \textbf{Quadratic OT} \end{cases}$$

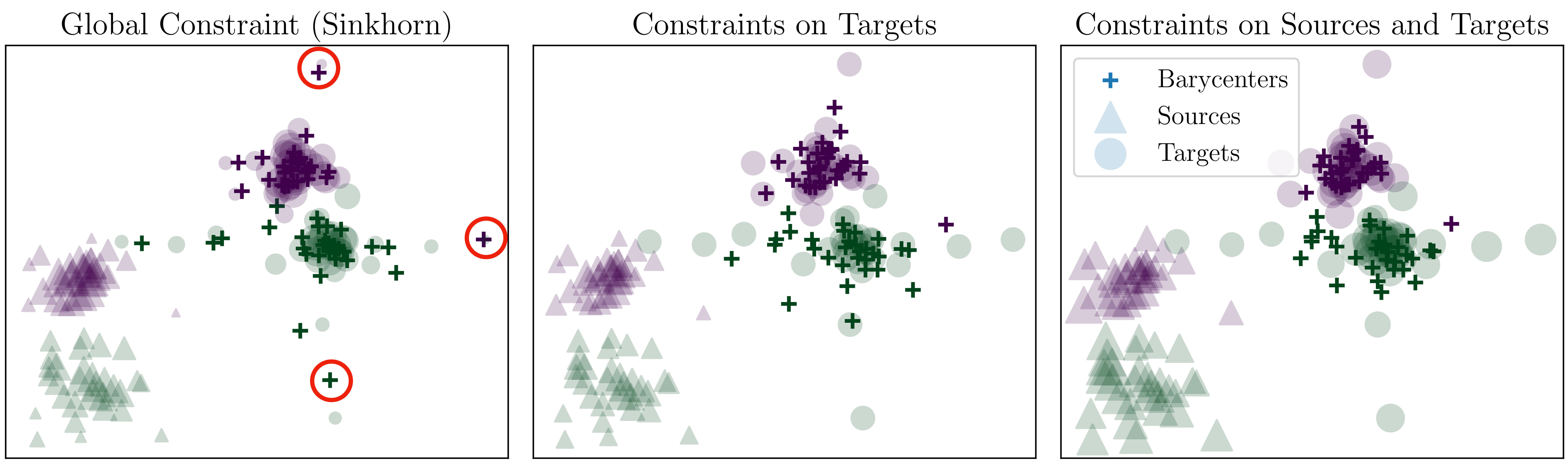
Diffusing mass for **outlier points** is costly, leading to **limited diffusion** for these points **under the global constraint**. To address this issue, we introduce **individual constraints for each point.**

OTARI

$$\min_{\mathbf{P} \in \Pi(\boldsymbol{a},\boldsymbol{b})} \langle \mathbf{P}, \mathbf{C} \rangle \quad \text{s.t.} \quad \forall i, \; \psi(\mathbf{P}_{i:}) \leq \eta_i \,.$$

Constraints can be applied on sources (as above), targets or both. OTARI problems can be solved using either **alternating ψ-Bregman projections** (Dykstra algorithm) or **dual ascent**.

Application to Domain Adaptation



- With traditional regularised OT, the barycentric mapping associated with an outlier is concentrated on the outlier.
- With OTARI, projections concentrate in high-density (thus more faithful) regions.

	OT	EOT	EOTARI-s	EOTARI-t	EOTARI-d
MNIST → USPS ($\xi = 30$)	53.1(5.4)	64.2(2.8)	65.0(5.3)	66.4(3.5)	**67.4**(**2.9**)
MNIST → USPS ($\xi = 300$)	53.1(5.4)	68.8(3.1)	70.8(4.2)	70.2(3.4)	**72.6**(**5.1**)
USPS → MNIST ($\xi = 30$)	59.1(4.9)	60.8(5.4)	61.6(4.4)	**62.6**(**3.0**)	61.0(4.7)
USPS → MNIST ($\xi = 300$)	59.1(4.9)	59.8(1.6)	61.0(2.3)	**61.6**(**3.0**)	58.8(2.3)

	OT	QOT	QOTARI-s	QOTARI-t	QOTARI-d
MNIST → USPS ($\xi = 30$)	53.1(5.4)	68.3(3.9)	68.3(3.6)	**69.3**(**4.7**)	68.1(4.6)
MNIST → USPS ($\xi = 300$)	53.1(5.4)	60.7(1.5)	**67.0**(**2.4**)	65.5(2.3)	65.8(2.5)
USPS → MNIST ($\xi = 30$)	59.1(4.9)	60.4(3.5)	**62.8**(**3.7**)	59.6(2.7)	61.6(3.1)
USPS → MNIST ($\xi = 300$)	59.1(4.9)	59.2(3.4)	60.1(3.0)	**62.0**(**3.7**)	61.5(3.8)